Cosmic Equation of State and Advanced LIGO Type Gravity Wave Experiments

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Abstract

Future generation of interferometric gravitational wave detectors is hoped to provide accurate measurements of the final stages of binary inspirals. The sources probed by such experiments are of extragalactic origin and the observed chirp mass is the intrinsic chirp mass multiplied by $(1 + z)$ where z is the redshift of the source. Moreover the luminosity distance is a direct observable is such experiments. This creates the possibility to establish a new kind of cosmological tests, supplementary to more standard ones.

Recent observations of distant type Ia supernovae light-curves suggest that the expansion of the universe has recently begun to accelerate. A popular explanation of present accelerating expansion of the universe is to assume that some part Ω_Q of the matter-energy density is in the form of dark component called "the quintessence" with the equation of state $p_Q = w\rho_Q$ with $w \geq -1$. In this paper we consider the predictions concerning observations of binary inspirals in future LIGO type interferometric experiments assuming a "quintessence cosmology". In particular we compute the expected redshift distributions of observed events in the a priori admissible range of parameters describing the equation of state for the quintessence. We find that this distribution has a robust dependence on the cosmic equation of state.

1 Introduction

Recent distance measurements from high-redshift type Ia supernovae [1, 2] suggest that the universe is presently accelerating its expansion. A popular explanation of this phenomenon is to assume that considerable amount $\Omega_Q \approx 70\%$ of the matter-energy density is in the form of dark component called "the quintessence" characterised by the equation of state $p_Q = w\rho_Q$ with $w \geq -1$ [3, 4, 5]. The evidence for spatially flat universe, reinforced by recent cosmic microwave background experiments BOOMERANG and MAXIMA [6, 7] calls for an extra unclustered dark component. Within the standard cold dark matter (CDM) scenario only about $0.2 < \Omega_{CDM} < 0.4$ can be clustered in order to be in agreement with galactic rotation curves, abundance of galaxy clusters, gravitational lensing or large scale velocity fields. Moreover the accelerated expansion of the universe can be achieved with extreme forms of matter. Hence this extra component (quintessence) should be similar to the cosmological constant but is allowed to have its own temporal dynamics. Many current models of dark matter in general and of quintessence in particular [8], invoke the concepts from particle physics. Particle physics, however, gives little guidance as to concrete models of quintessence. Therefore it has been proposed in [9] that future supernova surveys may allow reconstructing the quintessential equation of state. In this paper we shall contemplate the feasibility of constraining the cosmic equation of state from the gravitational wave experiments in a similar vein as proposed in [9].

Laser interferometric gravitational wave detectors developed under the projects LIGO, VIRGO and GEO600 are expected to perform a successful direct detection of the gravitational waves. Inspiralling neutron star (NS-NS) binaries are among the most promising astrophysical sources for this class of experiments [10]. Besides quite obvious benefits from seeing gravitational waves "in flesh" and providing valuable information about dynamical processes leading to their generation inspiralling binaries have one remarkable feature. Namely, the luminosity distance to a merging binary is a direct observable quantity easy to obtain from the waveforms. This circumstance made it possible to contemplate a possibility of accurate measurements of cosmological parameters such like the Hubble constant, or deceleration parameter [11, 12, 13]. In particular it was pointed out by Chernoff and Finn [11] how the catalogues of inspiral events can be utilised to make statistical inferences about the Universe. In the similar spirit we discuss in this paper the possibility to constrain the quintessence equation of state from the statistics of inspiral gravitational wave events.

2 Cosmological model

We shall consider a class of flat quintessential cosmological models. The spatially flat Universe has recently received a considerable observational support [14] from the measurements of the position of first acoustic peak at $l \approx 200$ in baloon experiments BOOMERANG [6] and MAXIMA [7]. This class is parametrized by two quantities: Ω_0 and Ω_Q, where $\Omega_0 = \rho/\rho_{cr} = \dfrac{8\pi G\rho_0}{3H_0^2}$ denotes the current matter density as a fraction of critical density for closing the Universe, Ω_Q is analogous fraction of critical density contained in the quintessence and these two sum up to the value one. The equation of state for the quintessence is assumed in a standard form: $p = w\rho$ where $w \geq -1$. This form of the equation of state is very general in the sense that it contains the well known constituents of the universe as special subclasses. For example $w = -1$ corresponds to the cosmological constant Λ, $w = 0$ – the dust matter, $w = -1/3$ – cosmic strings and $w = -2/3$ the domain walls.

Non Euclidean character of the space-time is reflected in distance measures. For the introduction to observational cosmology and the problems of distances in non-euclidean spaces (see e.g. [15]). In order to fix the notation for further use, let us introduce an auxiliary quantity $\mathcal{D}(z)$:

$$\mathcal{D}(z) = \sqrt{\Omega_0(1+z)^3 + \Omega_Q(1+z)^{3(1+w)}} \tag{1}$$

As it is well known, one can distinguish three types of distances:

(i) proper distance:

$$d_M(z) = \frac{c}{H_0}\int_0^z \frac{dw}{\mathcal{D}(w)} = \frac{d_H}{h}\int_0^z \frac{dw}{\mathcal{D}(w)} =: \frac{d_H}{h}\bar{d}_M(z) \tag{2}$$

(ii) angular distance:

$$d_A(z) = \frac{1}{1+z}d_M(z) = \frac{1}{1+z}\frac{d_H}{h}\bar{d}_M(z) \tag{3}$$

(iii) luminosity distance:

$$d_L(z) = (1+z)d_M(z) = (1+z)\frac{d_H}{h}\bar{d}_M(z) \tag{4}$$

As usually z denotes the redshift, h denotes the dimensionless Hubble constant i.e. $H_0 = h \times 100\ km/s\,Mpc$ and $d_H = 3. \times h^{-1}\ Gpc$ is the Hubble distance (radius of the Hubble horizon). The quantities with an overbar have been defined by factoring out the dependence on the Hubble constant from respective quantities. In the further discussion we will explore the following models:

$$(\Omega_0,\ \Omega_Q) = \{(0.2, 0.8);\ (0.3, 0.7);\ (0.4, 0.6)\}$$

with the w coefficient equal to $w = \{0, -0.2, -0.4, -0.6, -0.8, -1.\}$

¿From the observational point of view in the light of constraints from large scale structure and cosmic microwave background anisotropies, the 95% confidence interval estimates give $0.6 \le \Omega_Q \le 0.7$ and $-1. \le w < -0.6$ [16, 17].

However we retain the full spectrum of a priori possible quintessential equations of state in order to illustrate the discriminating power of the gravitational wave data discussed in this paper.

3 Redshift distribution of observed events

The gravity wave detector would register only those inspiral events for which the signal-to-noise ratio exceeded certain threshold value ρ_0 [11, 29] which is estimated as $\rho_0 = 8.$ for LIGO-type detectors. An intrinsic chirp mass $\mathcal{M}_0 = \mu^{3/5}M^{2/5}$,where μ and M denote the reduced and total mass, is the crucial observable quantity describing the inspiralling binary system. The observed chirp mass $\mathcal{M}(z) = (1+z)\mathcal{M}_0$ scales with the redshift and therefore can be used to determine the redshift to the source (there is strong evidence that the mass distribution of neutron stars in binary systems is sharply peaked around the value 1.4 $M_\odot$). Because the luminosity distance of a merging binary is a direct observable easily read off from the waveforms one has a possibility to determine the precise distance – redshift relation and hence to estimate the Hubble constant [29, 18]. For a given detector and a source the signal-to-noise ratio reads [29]:

$$\rho(z) = 8\Theta \frac{r_0}{d_L(z)} \left(\frac{\mathcal{M}(z)}{1.2\, M_\odot} \right)^{5/6} \zeta(f_{max}), \tag{5}$$

where r_0 is a charateristic distance scale, depending on detector's sensitivity, $r_0 \approx 355\ Mpc$ for advanced LIGO detectors, d_L is the luminosity distance to the source, $\zeta(f_{max})$ is a dimensionless function describing the overlap of the signal with detector's bandwidth.

The adiabatic inspiral signal terminates when the binary system reaches the innermost circular orbit (ICO). The corresponding orbital frequency is f_{ICO} and f_{max} corresponds to observed (i.e. redshifted) f_{ICO}

$$f_{max} = \frac{f_{ICO}}{1+z} = \frac{710\ Hz}{1+z} \left(\frac{2.8\ M_\odot}{M} \right), \tag{6}$$

so $f_{max} \sim 710\ Hz$ for neutron star binaries. It is argued that $\zeta(f_{max}) \approx 1$ for LIGO/VIRGO interferometers [11, 29].

Let us denote by $\dot{n}_0$ the local binary coalescing rate per unit comoving volume. One can use "the best guess" for local rate density $\dot{n}_0 \approx 9.9\ h\ 10^{-8}\ \ Mpc^{-3} yr^{-1}$ as inferred from the three observed binary pulsar systems that will coalesce in less than a Hubble time [19].
Source evolution over sample is usually parametrized by multiplying the coalescence rate by a factor $\eta(z) = (1+z)^D$, i.e. $\dot{n} = \dot{n}_0\ (1+z)^2\ \eta(z)$ where the $(1+z)^2$ factor accounts for the shrinking of volume with z and the time dilation of burst rate per unit time. The cosmological origin of gamma-ray bursts (GRBs) has been confirmed since discoveries of optical counterpart of GRB 970228 [20] and the measured emission-line redshift of $z = 0.853$ in GRB 970508 [21]. It has also been known for quite a long time that cosmological time dilation effects in BATSE catalogue suggest that the dimmest sources should be located at $z \approx 2$ [22]. Consequently several authors tackled the question of source evolution in the context of gamma-ray bursts. Early estimates of [23] and Piran [24] indicated that BATSE data could accommodate quite a large range of source density evoultion (from moderate negative to positive one). Later considerations by Horack et al. [25] indicated that if $z = 2$ is indeed the limiting redshift then a source population with a comoving rate density $n(z) \sim (1+z)^\beta$ with $1.5 \leq \beta \leq 2$ is compatible with BATSE data. Later on Totani [26] considered the source evolution effects and based his calculations on the realistic models of the cosmic star formation history in the context of NS-NS binary mergers. Comparison of the results with BATSE brightness distribution revealed that the NS-NS merger scenario of GRBs naturally leads to the rate evolution with $2 \leq \beta \leq 2.5$. We shall therefore take the source evolution effetcs into account in

our further considerations. One should stress, however that NS-NS merger scenario is by no means the unique explanation of gamma-ray bursts. Recently the so called collapsar model became popular [27]. The idea that at least some of gamma-ray bursts are related to the deaths of massive stars is supported by the observations of afterglows in GRB 970228 and GRB 980326 [?]. Therefore we will not prefer any specific value of evolution exponent D but instead we will try to illustrate how strongly and in which direction does the source evolution affect our ability to discriminate between different quintessential equations of state.

The relative orientation of the binary with respect to the detector is described by the factor Θ. This complex quantity cannot be measured nor assumed a priori. However, its probability density averaged over binaries and orientations has been calculated [29] and is given by a simple formula:

$$\begin{aligned} P_\Theta(\Theta) &= 5\Theta(4-\Theta)^3/256, \qquad \text{if} \quad 0 < \Theta < 4 \\ P_\Theta(\Theta) &= 0, \qquad \text{otherwise} \end{aligned} \tag{7}$$

The rate $\frac{d\dot{N}(>\rho_0)}{dz}$ at which we observe the inspiral events that originate in the redshift interval $[z,\ z+dz]$ is given by [30]:

$$\begin{aligned} \frac{d\dot{N}(>\rho_0)}{dz} &= \frac{\dot{n}_0}{1+z}\,\eta(z)\; 4\pi d_M^2 \frac{d}{dz} d_M(z)\; C_\Theta(x) = \\ &= 4\pi \left(\frac{d_H}{h}\right)^3 \frac{\dot{n}_0}{1+z}\,\frac{\bar{d}_M^2(z)}{\mathcal{D}(z)}\; C_\Theta(x) \end{aligned} \tag{8}$$

where $C_\Theta(x) = \int_x^\infty P_\Theta(\Theta) d\Theta$ denotes the probability that given detector registers inspiral event at redshift z_s with $\rho > \rho_0$. The quantity $C_\Theta(x)$ can be calculated as

$$\begin{aligned} C_\Theta(x) &= (1+x)(4-x)^4/256 \qquad \text{for} \;\; 0 \le x \le 4 \\ & \quad\, 0 \qquad \text{for} \;\; x > 4 \end{aligned} \tag{9}$$

where [31]:

$$\begin{aligned} x &= \frac{4}{h\,A}(1+z)^{7/6}\left[\frac{d_A(z)}{d_H/h}\right] = \\ &= \frac{4}{h\,A}(1+z)^{1/6}\;\bar{d}_M(z) \end{aligned} \tag{10}$$

and

$$A := 0.4733\,\left(\frac{8}{\rho_0}\right)\left(\frac{r_0}{355\;Mpc}\right)\,\left(\frac{\mathcal{M}_0}{1.2\;M_\odot}\right)^{5/6} \tag{11}$$

Figure 1 shows the expected detection rate of inspiralling events for the cosmological model with ($\Omega_0 = 0.3$, $\Omega_Q = 0.7$) assuming no source evolution and covering the full range of a priori possible quintessential equations of states. It has been obtained by numerical integration of the formula (8). The predictions for other realistic proportions of Ω_0 and Ω_Q are almost indistinguishable at the level of detection rates, so the Fig.1 is representative for the whole class of models considered. The effect of source evolution on the detection rate is summarised in Fig.2. For transparency only one member (corresponding to $w_q = -0.8$) of each family of curves (as in Fig.1) is shown for different values of the evolution exponent D.

The method of extracting the cosmological parameters advocated by Finn and Chernoff [29] makes use of the redshift distribution of observed events in a catalogue composed of observations with the signal-to-noise ratio greater than the threshold value ρ_0. Therefore it is important to find this distribution function for different quintessence models. The formula for the expected distribution of observed events in the source redshift can be easily obtained from the equation (8):

$$
\begin{aligned}
P(z, > \rho_0) &= \frac{1}{\dot{N}(> \rho_0)} \frac{d\dot{N}(> \rho_0)}{dt} = \\
&= \frac{4\pi}{\dot{N}(> \rho_0)} \left(\frac{d_H}{h}\right)^3 \frac{\dot{n}_0}{1+z}\, \eta(z)\, \frac{\bar{d}_M^2(z)}{\mathcal{D}(z)}\, C_\Theta(x) \qquad (12)
\end{aligned}
$$

The summary of numerical computations for the cosmological quintessence models considered based on the formulae (12) and (8) are given in figures Fig.3 and Fig.4. Fig.3 illustrates the $P(z, > \rho_0)$ distribution function for the ($\Omega_0 = 0.3$, $\Omega_Q = 0.7$) cosmological model with different quintessential equations of state. For the purpose of obtaining the Figures 3 and 4 we have assumed the dimensionless Hubble constant equal to $h = 0.65$ as suggested by independent observational evidence (e.g. SNe Ia in HST project [32] or multiple image quasar systems [33]). On Fig.4 the distribution functions for different cosmological models with the quintessence field with $w = -0.8$ have been plotted together. Fig.5 shows the distribution functions for different evolutionary exponent in the ($\Omega_0 = 0.3$, $\Omega_Q = 0.7$) model with $w = -0.8$.

4 Results and discussion

It is clear from Figure 1 that different quintessential cosmologies (singled out by w parameter in the equation of state) give different predictions for annual inspiral event rate to be observed by future interferometric experiments. Unfortunately , this difference is too small to be of observational importance. Moreover, as already pointed out, there exists a degeneracy in terms of cosmological models (labelled by the value of Ω_0 and Ω_Q). There is however a difference between detection rates corresponding to different values of evolutionary exponents as displayed in Figure 2.

Figure 3 shows that there is a noticeable difference in predicted event redshift distribution functions $P(z, > \rho_0)$ for different values of the cosmic equation of state within given cosmological model (labelled by the values of Ω_0 and Ω_Q). The spread between different cosmological models for a given quintessence equation of state is much smaller as seen from the Fig.4. This is a reflection of above mentioned effective degeneracy with respect to values of Ω parameters. Hopefully this degeneracy can be broken by independent estimates of Ω_0 and Ω_Q parameters in other studies (cluster baryons estimates, Lyα forest surveys, large scale structure or CMBR). The spread of redshift distribution functions attributed to evolutionary effects is smaller as shown in Fig. 5 and has a slightly different character - the distribution function is shifted toward increasing redshifts when the evolutionary exponent changes from positive to negative value. This may to some extent mimic the effect of cosmic equation of state, but it should in principle be possible to disentangle - at least to a certain degree from the complementary information about the detection rates. As can be seen from Fig.2 the magnitudes of observed event rates for different evolutionary exponents D are clearly distinct, at least for the range of the Hubble constant suggested by independent cosmological evidence [34].

The redshift distribution $P(z, > \rho_0)$ is in fact inferred from observed chirp mass distribution. Therefore it can in principle be distorted by the intrinsic chirp mass distribution. Theoretical studies of the neutron star formation suggest that masses of nascent neutron stars do not vary much with either mass or composition of the progenitor [29]. Also the mass estimates of observed binary pulsars suggest that there are good reasons to assume a negligible spread of intrinsic chirp mass (as it was done in the present paper). Moreover any intrinsic distribution of mass would be expected as symmetric, whereas the redshift distribution (of cosmological origin) has certain amount of asymmetry.

In conclusion one can hope that the catalogues of inspiral events gathered in future gravitational waves experiments can provide helpful information about the quintessence equation of state complementary to that obtained by other techniques. Even though the most straightforward way of making inference about cosmic equation of state would come from future supernovae surveys it would be good to have in mind alternative ways of reaching the same goal such as the one proposed in the present paper.

Figure 1

The detection rate prediction for the advanced gravity wave detectors i.e. with signal-to-noise threshold $\rho_0 = 8.$ and probing distance $r_0 = 355\ Mpc.$ corresponding to quintessence cosmology with different equations of state.

Figure 2

The detection rate prediction for the advanced gravity wave detectors i.e. with signal-to-noise threshold $\rho_0 = 8.$ and probing distance $r_0 = 355\ Mpc.$ corresponding to $\Omega_0 = 0.3,\ \ \Omega_Q = 0.7$ quintessence cosmology with $w_q = -0.8$ for different values of evolutionary exponent D.

Figure 3

Redshift distribution of observed events in the cosmological model with $\Omega_0 = 0.3,\ \ \Omega_Q = 0.7$ for different quintessential equations of state.

Figure 4

Redshift distribution of observed events in the cosmological quintessence model with $w = -0.8$ Different cosmological models have been plotted collectively.

Figure 5

Redshift distribution of observed events in the cosmological model with $\Omega_0 = 0.3,\ \ \Omega_Q = 0.7$ with $w_q = -0.8$ quintessence for different values of evolutionary exponents D.

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